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Structures of Neutrino Flavor Mixing Matrix and Neutrino Oscillations at CHORUS and NOMAD

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ABSTRACT

We have studied structures of the neutrino flavor mixing matrix focusing on the neutrino oscillations at CHORUS and NOMAD as well as the one at LSND(or KAR-MEN). We have assumed two typical neutrino mass hierarchies $m_3 \simeq m_2 \gg m_1$ and $m_3 \gg m_2 \gg m_1$ (or $\simeq m_1$). Taking into account the see-saw mechanism of neutrino masses, the reasonable neutrino flavor mixing patterns have been discussed. The observation of the neutrino oscillation at CHORUS and NOMAD presents us the important constraint for the structure of the neutrino flavor mixing matrix. The atmospheric neutrino anomaly has been discussed in relation to the CHORUS and NOMAD experiments.

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1. Introduction

Neutrino flavor oscillations are important phenomena to search, at low energy, for physics beyond the Standard Model of the electroweak interaction, and to get information on very high energy scales via the see-saw mechanism of the neutrino masses[1]. However, the only possible evidences for neutrino oscillations originate from the natural beams: the solar neutrinos[2] and the atmospheric neutrinos[3,4,5]. In the near future, data from the accelerator and reactor neutrino experiments will be available. CHORUS[6] is expected to present the first result soon. The first long base line reactor experiment CHOOZ[7] will begin to operate. These experiments may resolve neutrino puzzles.

The tentative indication has been already given by the LSND experiment[8]. It was reported that an excess of 9 electron events was observed at LSND. If these events are due to the neutrino oscillation, the average $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation transition probability is equal to $0.34^{+0.20}_{-0.18} \pm 0.07\%$ [8,9]. KARMEN experiment[10] is also searching for the $\nu_\mu \rightarrow \nu_e$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$) oscillation as well as LSND. The CHORUS[6] and NOMAD[11] experiments at CERN are looking for the $\nu_\mu \rightarrow \nu_\tau$ oscillation. The most powerful reactor experiments searching for the neutrino oscillation are Bugey[12] and Krasnoyarsk[13] at present. They provide excluded regions in $(\sin^2 2\theta, \Delta m^2)$ parameter space by non-observation of the neutrino oscillation.

One expects to extract the neutrino flavor mixing matrix from the data of neutrino flavor oscillations. In this paper, we study structures of the neutrino flavor mixing matrix focusing on the neutrino oscillations at CHORUS[6] and NOMAD[11] as well as the one at LSND(or KARMEN). We find that the observation of the neutrino oscillation at CHORUS and NOMAD presents us the important constraint for the structure of the neutrino flavor mixing matrix.

It is emphasized that there are only two hierarchical mass difference scales Δm^2 in the three-flavor mixing scheme without introducing sterile neutrinos. The neutrino with the mass $1 \sim 7\text{eV}$ is a candidate of the dark matter. In particular, the cold+hot dark matter model has been shown to agree well with cosmological observations, galaxies formation[14]. If we take this mass scale for the highest one in the neutrino masses, the other mass scale is either the atmospheric neutrino mass scale $\Delta m^2 \simeq 10^{-2}\text{eV}^2$ or the solar neutrino one $\Delta m^2 \simeq 10^{-5} \sim 10^{-6}\text{eV}^2$. In our analyses, one neutrino mass scale is taken to be $1 \sim 7\text{eV}$, below which CHORUS and NOMAD experiments will not be fruitful[6,11]. The other scale is fixed to be the atmospheric neutrino one. The solar neutrino problem will be also discussed briefly in section 4. By using the recent data from the accelerator and reactor neutrino experiments[15-18], we investigate the probability of the $\nu_\mu \rightarrow \nu_\tau$ oscillation at CHORUS and NOMAD. Based on these results, we discuss the structure of the neutrino flavor mixing matrix.

In section 2, we give the formulation of the neutrino oscillations, and we discuss constraints from present accelerator and reactor data in section 3. In section 4, we present numerical analyses at CHORUS and NOMAD, and discuss structures of the neutrino flavor mixing matrix. Section 5 is devoted to conclusions.

2. Formulation of Neutrino Oscillations

Recent analyses of the three flavor neutrino oscillation are helpful for our work[19-23]. In particular, the quantitative results by Bilenky et al.[20] and Fogli, Lisi and Scioscia[22] are the useful guide for our analyses although we concentrate mainly on the CHORUS and NOMAD results, which will be presented soon.

The oscillation probability of neutrinos of energy E_ν after traversing the distance

L can be written as

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E_\nu}\right), \quad (1)$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$ is defined, and $U_{\alpha i}$ denote the elements of the 3×3 neutrino flavor mixing matrix, in which α and i refer to the flavor eigenstate and the mass eigenstate, respectively. Since we neglect the CP violation, the mixing parameters $U_{\alpha i}$ are real in our analyses. We assume two typical hierarchical relations $\Delta m_{31}^2 \gg \Delta m_{32}^2$ and $\Delta m_{31}^2 \gg \Delta m_{21}^2$ in order to guarantee two different mass scales. The former relation corresponds to $m_3 \simeq m_2 \gg m_1$ and the latter one to $m_3 \gg m_2 \gg m_1$ (or $\simeq m_1$). In our analyses, the highest neutrino mass scale is taken to be $\Delta m_{31}^2 = 1 \sim 50 \text{eV}^2$, which is appropriate for the cosmological hot dark matter.

The value of L/E_ν is fixed in each experiment. In our following analyses, for example, $L = 800 \text{m}$ and $E_\nu = 30 \text{GeV}$ are taken for the $\nu_\mu \rightarrow \nu_\tau$ experiment at CHORUS, and $L = 30 \text{m}$ and $E_\nu = 36 \sim 60 \text{GeV}$ for the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ experiment at LSND. For the atmospheric neutrino, $L = 10 \sim 10^4 \text{km}$ and $E_\nu = 0.1 \sim 10 \text{GeV}$ are expected. The approximate formulae of the oscillation probabilities are given for the accelerator and the reactor as follows. For the case of $\Delta m_{31}^2 \gg \Delta m_{32}^2$, those are given as

$$P(\nu_\mu \rightarrow \nu_e) = 4U_{e1}^2 U_{\mu 1}^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right), \quad (2)$$

$$P(\nu_\mu \rightarrow \nu_\tau) = 4U_{\mu 1}^2 U_{\tau 1}^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right), \quad (3)$$

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4U_{\alpha 1}^2 (1 - U_{\alpha 1}^2) \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right). \quad (4)$$

On the other hand, for the case of $\Delta m_{31}^2 \gg \Delta m_{21}^2$, those are written as

$$P(\nu_\mu \rightarrow \nu_e) = 4U_{e3}^2 U_{\mu 3}^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right), \quad (5)$$

$$P(\nu_\mu \rightarrow \nu_\tau) = 4U_{\mu 3}^2 U_{\tau 3}^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right), \quad (6)$$

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4U_{\alpha 3}^2(1 - U_{\alpha 3}^2) \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right). \quad (7)$$

If the atmospheric neutrino anomaly is attributed to the $\nu_\mu \rightarrow \nu_\tau$ oscillation, the relevant formulae are given instead of eqs.(3) and (6),

$$P(\nu_\mu \rightarrow \nu_\tau) = -4U_{\mu 2}U_{\tau 2}U_{\mu 3}U_{\tau 3} \sin^2\left(\frac{\Delta m_{32}^2 L}{4E_\nu}\right) + 2U_{\mu 1}^2 U_{\tau 1}^2, \quad (8)$$

and

$$P(\nu_\mu \rightarrow \nu_\tau) = -4U_{\mu 1}U_{\tau 1}U_{\mu 2}U_{\tau 2} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E_\nu}\right) + 2U_{\mu 3}^2 U_{\tau 3}^2, \quad (9)$$

respectively since L/E_ν of the atmospheric neutrino is much larger than L/E_ν of the accelerators and the reactors. The atmospheric neutrino anomaly may be due to the $\nu_\mu \rightarrow \nu_e$ oscillation. Then, instead of eqs.(2) and (5), the probabilities are given as

$$P(\nu_\mu \rightarrow \nu_e) = -4U_{e 2}U_{\mu 2}U_{e 3}U_{\mu 3} \sin^2\left(\frac{\Delta m_{32}^2 L}{4E_\nu}\right) + 2U_{e 1}^2 U_{\mu 1}^2, \quad (10)$$

and

$$P(\nu_\mu \rightarrow \nu_e) = -4U_{e 1}U_{\mu 1}U_{e 2}U_{\mu 2} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E_\nu}\right) + 2U_{e 3}^2 U_{\mu 3}^2, \quad (11)$$

respectively. Eqs.(2)~(11) are used in the following analyses.

3. Constraints from Present Accelerator and Reactor Data

Let us begin with discussing constraints of the accelerator and reactor disappearance experiments. Since no indications in favor of neutrino oscillations were found in these experiments, we only get the allowed regions in $(U_{\alpha i}^2, \Delta m_{31}^2)$ parameter space. The recent Bugey reactor experiment[12] and the CDHS[15] and CCFR[16] accelerator experiments give the bounds for the neutrino mixing parameters at the fixed value of Δm_{31}^2 . We follow the analyses given by Bilenky et al.[20].

From eqs.(4) and (7) the mixing parameters can be expressed in terms of the oscillation probabilities as[20]

$$U_{\alpha i}^2 = \frac{1}{2}(1 \pm \sqrt{1 - B_{\nu_\alpha \nu_\alpha}}) , \quad (12)$$

with

$$B_{\nu_\alpha \nu_\alpha} = \frac{1 - P(\nu_\alpha \rightarrow \nu_\alpha)}{\sin^2(\frac{\Delta m_{31}^2 L}{4E_\nu}} , \quad (13)$$

where $\alpha = e, \mu$ and $i = 1, 3$. Therefore, the parameters $U_{\alpha i}^2$ at the fixed value of Δm_{31}^2 should satisfy one of the following inequalities:

$$U_{\alpha i}^2 \geq \frac{1}{2}(1 + \sqrt{1 - B_{\nu_\alpha \nu_\alpha}}) \equiv a_\alpha^{(+)} , \quad \text{or} \quad U_{\alpha i}^2 \leq \frac{1}{2}(1 - \sqrt{1 - B_{\nu_\alpha \nu_\alpha}}) \equiv a_\alpha^{(-)} . \quad (14)$$

In Table 1, we show the values of $a_e^{(\pm)}$ and $a_\mu^{(\pm)}$, which were obtained[20] from the negative results of the Bugey[12], CDHS[15] and CCFR[16] experiments, for the typical values of $\Delta m_{31}^2 = 1, 4, 6, 30, 50 \text{eV}^2$.

Table 1

From eq.(14) it is noticed there are three allowed regions of U_{ei}^2 and $U_{\mu i}^2$ as follows[20,22]:

$$\begin{aligned} (A) \quad & U_{ei}^2 \geq a_e^{(+)} , \quad U_{\mu i}^2 \leq a_\mu^{(-)} , \\ (B) \quad & U_{ei}^2 \leq a_e^{(-)} , \quad U_{\mu i}^2 \leq a_\mu^{(-)} , \\ (C) \quad & U_{ei}^2 \leq a_e^{(-)} , \quad U_{\mu i}^2 \geq a_\mu^{(+)} , \end{aligned} \quad (15)$$

where $i = 1$ or 3 corresponding to the neutrino mass hierarchies. In addition to these constraints, we should take account of the constraints by the E531[17] and E776[18] experimental data. In some cases, these constraints become important.

4. Numerical Analyses at CHORUS and NOMAD

We study the $\nu_\mu \rightarrow \nu_\tau$ oscillation at CHORUS(NOMAD) under the constraints of other experiments. In particular, we discuss it in relation to the $\nu_\mu \rightarrow \nu_e$ oscillation at LSND(KARMEN) and the $\nu_\mu \rightarrow \nu_X$ oscillation of the atmospheric neutrino. We assume two hierarchical relations $\Delta m_{31}^2 \gg \Delta m_{32}^2$ and $\Delta m_{31}^2 \gg \Delta m_{21}^2$, ie., the mass hierarchies such as $m_3 \simeq m_2 \gg m_1$ and $m_3 \gg m_2 \gg m_1$ (or $\simeq m_1$). The former hierarchy was suggested by investigating the hot dark matter in ref.[14], where $m_3 \simeq m_2 \simeq 2.4\text{eV}$, ie., $\Delta m_{31}^2 \simeq 6\text{eV}^2$ is preferred. We call this case hierarchy I. In the latter case, $m_3 \leq 7\text{eV}$ is expected as the candidate of the hot dark matter although this value depends on the Hubble parameter. The case is called hierarchy II. We analyze the neutrino oscillations for both hierarchies I and II in the cases (A), (B) and (C) given in eq.(15).

Let us start with discussing the case (A), in which we have $U_{ei}^2 \geq a_e^{(+)}$ and $U_{\mu i}^2 \leq a_\mu^{(-)}$. For the hierarchy I, eqs.(2), (3), (4), (8), (10) are adopted for the relevant neutrino oscillations. The magnitude of the $\nu_\mu \rightarrow \nu_e$ oscillation depends on $U_{\mu 1}^2$ because U_{e1}^2 is close to 1. Since $P(\nu_\mu \rightarrow \nu_e) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ is guaranteed in the CP conserved limit, we express the oscillation probability at LSND as $P_{\text{LSND}} \equiv P(\nu_\mu \rightarrow \nu_e)$, while P_{CHORUS} denotes $P(\nu_\mu \rightarrow \nu_\tau)$ at CHORUS. Then, we have the following relation between P_{CHORUS} and P_{LSND} :

$$P_{\text{CHORUS}} \simeq P_{\text{LSND}} \times U_{\tau 1}^2 \sin^2(\Delta m_{31}^2 \frac{L}{4E_\nu})_{\text{CHORUS}} / \sin^2(\Delta m_{31}^2 \frac{L}{4E_\nu})_{\text{LSND}} . \quad (16)$$

If we take $\Delta m_{31}^2 \simeq 6\text{eV}^2$, we can estimate the upper bound of P_{CHORUS} by using the LSND result. Even if the reported LSND events[8] are due to the neutrino oscillations, P_{LSND} should be lower than 1.9×10^{-3} which is derived from the upper bound of E776[18]. By using this bound, we obtain $P_{\text{CHORUS}} \leq 3 \times 10^{-6}$, which is hopeless to be probed by CHORUS and NOMAD.

In the case of the hierarchy II, eqs.(5), (6), (7), (9), (11) are adopted for the

neutrino oscillations. The mixing parameters $U_{\alpha 3}^2$ play an important role instead of $U_{\alpha 1}^2$. The numerical discussion is completely parallel to the one in the hierarchy I. By using the E776 upper bound[18], we get $P_{\text{CHORUS}} \leq 3 \times 10^{-6}(\Delta m_{31}^2 = 6\text{eV}^2) \sim 1.2 \times 10^{-4}(\Delta m_{31}^2 = 50\text{eV}^2)$, which are out of the sensitivity at CHORUS and NOMAD.

The structures of the neutrino flavor mixing matrix \mathbf{U} are different between the hierarchies I and II. The mixing matrix in the hierarchies I is written as

$$\mathbf{U} \simeq \begin{pmatrix} 1 & \epsilon_3 & \epsilon_4 \\ \epsilon_1 & U_{\mu 2} & U_{\mu 3} \\ \epsilon_2 & U_{\tau 2} & U_{\tau 3} \end{pmatrix}, \quad (17)$$

where $\epsilon_i (i = 1 \sim 4)$ are tiny numbers. As seen in eq.(8), the atomospheric neutrino anomaly could be solved by the large $\nu_\mu \rightarrow \nu_\tau$ oscillation by taking $\Delta m_{32}^2 \simeq 0.01\text{eV}^2$. On the other hand, it is impossible to explain the anomaly by the large $\nu_\mu \rightarrow \nu_e$ oscillation in eq.(10) because $U_{e2} = \epsilon_3$ and $U_{e3} = \epsilon_4$ are very small. The mixing matrix

$$\mathbf{U} \simeq \begin{pmatrix} 1 & \epsilon_3 & \epsilon_4 \\ \epsilon_1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \epsilon_2 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (18)$$

is consistent with the atomospheric neutrino anomaly in Kamiokande[5]. Thus the hierarchy I could be consistent with the atomospheric neutrino anomaly although we cannot expect signals of the neutrino oscillation at CHORUS and NOMAD.

For the hierarchy II, the mixing matrix is given as

$$\mathbf{U} \simeq \begin{pmatrix} \epsilon_1 & \epsilon_2 & 1 \\ U_{\mu 1} & U_{\mu 2} & \epsilon_3 \\ U_{\tau 1} & U_{\tau 2} & \epsilon_4 \end{pmatrix}. \quad (19)$$

The atomospheric neutrino anomaly is also explained by the large $\nu_\mu \rightarrow \nu_\tau$ oscillation with $\Delta m_{21}^2 \simeq 0.01\text{eV}^2$ [5] in eq.(9). Then, the mixing matrix is expected to be

$$\mathbf{U} \simeq \begin{pmatrix} \epsilon_1 & \epsilon_2 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \epsilon_3 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \epsilon_4 \end{pmatrix}. \quad (20)$$

This matrix structure in the hierarchy II mean that the heaviest neutrino is almost the flavor "e" neutrino. In this case, the neutrino mass hierarchy is a inverse one

compared with the generation hierarchy of quark masses. The inverse mass hierarchy of ν_3 (1st-generation) and ν_1 (3rd-generation) should be derived from the structure of the right-handed Majorana mass matrix in the see-saw mechanism of the neutrino masses[1]. Since $\Delta m_{31}^2/\Delta m_{21}^2 \simeq 10^{2\sim 4}$, the following relation should be satisfied in order to guarantee the inverse neutrino masses,

$$m_3 \geq 10^{1\sim 2} \times m_1, \quad \text{with} \quad m_3 = \frac{m_1^{D2}}{M_1}, \quad m_1 = \frac{m_3^{D2}}{M_3}, \quad (21)$$

where $m_1^D(m_3^D)$ and $M_1(M_3)$ are the Dirac and Majorana masses of the first(third) generation, respectively. Strictly speaking, this condition is not an exact one because of the off-diagonal elements in the mass matrices. However, we can safely neglect the contribution of the off diagonal elements since the mixings $U_{\mu 3}$ and $U_{\tau 3}$ are very small. From eq.(21) we get

$$\frac{M_3}{M_1} \geq 10^{1\sim 2} \times \left(\frac{m_3^D}{m_1^D} \right)^2 \simeq 10^{10\sim 11}, \quad (22)$$

where the t-quark and u-quark masses are taken for m_3^D and m_1^D , respectively. This huge mass hierarchy of the Majorana masses seems to be unnatural. We conclude that the case (A) with the hierarchy II is ruled out because of the inverse hierarchy of the neutrino masses.

In the case (B), we get $U_{\tau i}^2 \simeq 1$ since both U_{ei}^2 and $U_{\mu i}^2$ are very small as seen in eq.(15). Then, P_{CHORUS} depends on only $U_{\mu i}^2$, which is constrained by the E531[17] bound of the $\nu_\mu \rightarrow \nu_\tau$ oscillation. On the other hand, P_{LSND} is suppressed because both U_{ei}^2 and $U_{\mu i}^2$ are very small. In the hierarchy I with $\Delta m_{31}^2 \simeq 6\text{eV}^2$, we get the bounds

$$P_{\text{CHORUS}} \leq 9.8 \times 10^{-4}, \quad P_{\text{LSND}} \leq 5.1 \times 10^{-4}, \quad (23)$$

where we used $U_{\mu 1}^2 \leq 6.0 \times 10^{-3}$ at E531[17] and the $U_{e1}^2 \leq 0.036$ at Bugey[12] in Table 1. Obviously, CHORUS and MOMAD are expected to observe the neutrino oscillation but our obtained P_{LSND} contradicts to the reported LSND probability, $0.34_{-0.18}^{+0.20} \pm 0.07\%$ [8].

In the hierarchy II, U_{e3}^2 and $U_{\mu3}^2$ are relevant mixing parameters instead of U_{e1}^2 and $U_{\mu1}^2$. These mixing parameters are also constrained by the E531, CDHS and Bugey experiments. We show in fig.1 the upper bounds of P_{CHORUS} and P_{LSND} with the limit of the sensitivity at CHORUS for $\Delta m_{31}^2 = 1 \sim 50\text{eV}^2$. The solid curve denotes the upper bound of P_{CHORUS} , which is given by the E531 bound of $U_{\mu3}^2$ [17] for $\Delta m_{31}^2 = 4 \sim 50\text{eV}^2$ and by the CDHS bound[15] for $\Delta m_{31}^2 = 1 \sim 4\text{eV}^2$. The dashed curve denotes the upper bound of P_{LSND} , which is given by E531, CDHS and Bugey data. As shown in fig.1, CHORUS and MOMAD are hopeful to observe the neutrino oscillation, however, our predicted upper bound of LSND is below the recent reported LSND events[8].

fig. 1

As discussed in the case (A), the structures of the neutrino flavor mixing matrix \mathbf{U} are also different between the hierarchies I and II in the case (B). The mixing matrices being consistent with the atmospheric neutrino anomaly[5] are expected to be

$$\mathbf{U} \simeq \begin{pmatrix} \epsilon_1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \epsilon_2 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & \epsilon_3 & \epsilon_4 \end{pmatrix}, \quad (24)$$

for the hierarchy I, and

$$\mathbf{U} \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \epsilon_1 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \epsilon_2 \\ \epsilon_3 & \epsilon_4 & 1 \end{pmatrix}, \quad (25)$$

for the hierarchy II, respectively. It is noticed that, in the case (B), the atmospheric neutrino anomaly could occur due to the large $\nu_\mu \rightarrow \nu_e$ oscillation as seen in eqs.(10) and (11). In the hierarchy I, the generation hierarchy of neutrino masses is the inverse one since " τ " neutrino is the lightest one, while it is an ordinary one in the hierarchy II. The inverse neutrino mass hierarchy leads to the unnatural huge mass difference of the Majorana masses $O(10^{10 \sim 11})$, as discussed in the case (A). Therefore, the case (B) with the hierarchy I is ruled out.

One may consider that the case (B) with hierarchy II is a natural case consistent with the atmospheric neutrino anomaly. However, the large $\nu_\mu - \nu_e$ mixing is excluded by the reactor experiments at Bugey[12] and Krasnoyarsk[13]. The mixing angle is constrained such as $\sin^2 2\theta_{e\mu} \leq 0.7$ in the case of $\Delta m_{21}^2 = 10^{-2} \text{eV}^2$ in the reactor experiments while the data of the atmospheric neutrino anomaly in Kamiokande[5] suggests $\Delta m_{21}^2 = 7 \times 10^{-3} \sim 8 \times 10^{-2} \text{eV}^2$ and $\sin^2 2\theta_{e\mu} = 0.6 \sim 1$ for the $\nu_\mu \rightarrow \nu_e$ oscillation. The overlap region is rather small such as $\sin^2 2\theta_{e\mu} = 0.6 \sim 0.7$. The new reactor experiments will soon give the severer constraint for the $\nu_\mu - \nu_e$ mixing.

The case (C) is in the region of $U_{\mu i}^2 \simeq 1$. Then, P_{CHORUS} depends on only $U_{\tau i}^2$, which is constrained by the E531[17] bound of the $\nu_\mu \rightarrow \nu_\tau$ oscillation. On the other hand, P_{LSND} depends on U_{ei}^2 , which is constrained by the E776[18] bound of the $\nu_\mu \rightarrow \nu_e$ oscillation. Hence, there is no relation between P_{CHORUS} and P_{LSND} for the present. In the hierarchy I with $\Delta m_{13}^2 \simeq 6 \text{eV}^2$, we get the bounds

$$P_{\text{CHORUS}} \leq 9.8 \times 10^{-4} , \quad P_{\text{LSND}} \leq 1.9 \times 10^{-3} , \quad (26)$$

where we used the E531 bound $U_{\tau 1}^2 \leq 6.0 \times 10^{-3}$ and the E776 bound $U_{e1}^2 \leq 5.2 \times 10^{-4}$. CHORUS and MOMAD are expected to observe the neutrino oscillation. Of course, the reported LSND events[8] are still consistent with our result.

In the the hierarchy II, U_{e3}^2 and $U_{\tau 3}^2$ are relevant mixing parameters instead of U_{e1}^2 and $U_{\tau 1}^2$. These parameters are bounded by the E531, CDHS and E776 experiments. We show the upper bounds of P_{CHORUS} and P_{LSND} in fig.2, in which notations are same as in fig.1. It is found that CHORUS and MOMAD are also hopeful to observe the neutrino oscillation. The prediction is also consistent with the reported LSND events[8].

fig. 2

The structures of the neutrino mixing matrix are given as

$$\mathbf{U} \simeq \begin{pmatrix} \epsilon_1 & U_{e2} & U_{e3} \\ 1 & \epsilon_2 & \epsilon_3 \\ \epsilon_4 & U_{\tau 2} & U_{\tau 3} \end{pmatrix}, \quad (27)$$

for the hierarchy I, and

$$\mathbf{U} \simeq \begin{pmatrix} U_{e1} & U_{e2} & \epsilon_1 \\ \epsilon_2 & \epsilon_3 & 1 \\ U_{\tau 1} & U_{\tau 2} & \epsilon_4 \end{pmatrix}, \quad (28)$$

for the hierarchy II. In both hierarchies, we cannot explain the atmospheric neutrino anomaly by the neutrino oscillations because ϵ_2 and ϵ_3 remain to be small due to the unitarity. Since the flavor " μ " couples to the lightest neutrino strongly in the hierarchy I, the masses of ν_1 and ν_2 are inverse compared with the generation of quarks. On the other hand, the masses of ν_2 and ν_3 are inverse in the hierarchy II as the flavor " μ " couples to the heaviest neutrino. The magnitude of the Majorana mass hierarchy is estimated in the same way as in eqs.(21) and (22) by using the top-quark, c-quark and u-quark masses for m_3^D , m_2^D and m_1^D , respectively. The mass hierarchy of $M_1 \sim M_3$ is roughly larger than $O(10^6)$, which may be allowed to build a natural model.

We summarize in Table 2 the predicted upper bound of P_{CHORUS} and P_{LSND} in the case of $\Delta m_{31}^2 = 6\text{eV}^2$, the situation of the atmospheric neutrino anomaly, and the neutrino mass hierarchy for each case.

Table 2

As the results of our analyses of cases (A), (B) and (C), we give remarks on the atmospheric neutrino anomaly. The anomaly could be explained by the ν_μ oscillation in the cases (A) and (B). It is emphasized that either $\nu_\mu \rightarrow \nu_\tau$ or $\nu_\mu \rightarrow \nu_e$ modes could be operative in the atmospheric neutrino. There is no solution that both modes are dominant. In our analyses, we fixed the second mass scale as the atmospheric neutrino mass scale. Therefore, we cannot explain the solar neutrino problem by the

neutrino oscillation without introducing sterile neutrino[24,25]. On the other hand, if we abandon the possibility of solving the atmospheric neutrino anomaly by the neutrino oscillation, we can take the second mass scale as the solar neutrino mass scale. Taking the MSW solution[26], the solar neutrino problem could be reconciled with our analyses except for one case, which is the case (A) with the hierarchy II as pointed out by Bilenky et al.[20]. Since we have $U_{e3} \simeq 1$ in this exceptional case, the survival probability of the solar neutrinos is too large to be consistent with the data of GALLEX and SAGE, which have shown less neutrino deficit than the Homestake and Kamiokande experiments[2].

5. Conclusions

We have studied structures of the neutrino flavor mixing matrix focusing on the neutrino oscillations at CHORUS and NOMAD as well as the one at LSND(or KARMEN). We assumed the typical mass hierarchies $m_3 \simeq m_2 \gg m_1$ (hierarchy I) and $m_3 \gg m_2 \gg m_1$ (or $\simeq m_1$)(hierarchy II). Taking into account the see-saw mechanism[1] of neutrino masses, the reasonable neutrino flavor mixing patterns have been discussed. In the case (A), only the hierarchy I gives the reasonable neutrino flavor mixing matrix. Then, CHORUS and NOMAD are hopeless to observe the neutrino oscillations although LSND(KARMEN) could observe the oscillation. The atmospheric neutrino anomaly could be explained due to the large $\nu_\mu \rightarrow \nu_\tau$ oscillation. In the case (B), the hierarchy II gives the reasonable neutrino flavor mixing matrix. CHORUS and NOMAD are hopeful to observe the evidence of the neutrino oscillations, but the expected upper bound of the LSND(KARMEN) experiment is much smaller than the present experimental sensitivity. We have a small chance to explain the atmospheric neutrino anomaly by the large $\nu_\mu - \nu_e$ mixing $\sin^2 2\theta_{e\mu} = 0.6 \sim 0.7$. The new reactor experiments will soon check this case. Although the case (C) leads to the inverse hier-

archy for the Majorana masses, it may be still natural to build a model. In this case, both CHORUS(NOMAD) and LSND(KARMEN) could observe the evidence of the neutrino oscillations, but the atmospheric neutrino anomaly could not be explained due to the neutrino oscillations.

If CHORUS will observe the signal of the neutrino oscillation, the case (B) with the hierarchy II and the case (C) are the reasonable cases as seen in Table 2. Then, we have to wait for more sensitive experiments at LSND or KARMEN in order to determine the value of the heaviest neutrino mass. The experimental study of the atmospheric neutrino anomaly in Super-Kamiokande is also important to decide the favorable case. Thus, the observation of the neutrino oscillation at CHORUS and NOMAD presents us the important constraint for the structure of the neutrino flavor mixing matrix, which is very useful guide to go beyond the Standard Model of the quark-lepton unification.

What can we learn if CHORUS and NOMAD will observe no signals of the neutrino oscillation? The case (A) with hierarchy I is most reasonable case if the atmospheric neutrino anomaly is caused by the large flavor mixing. Otherwise, we will need more detailed studies by using the improved limits of $U_{\mu i}$ and $U_{\tau i}(i = 1, 3)$ by CHORUS and NOMAD in the case (C).

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Figure Captions

figure 1:

Upper bounds of P_{CHORUS} and P_{LSND} versus Δm_{31}^2 in the case (B) with the hierarchy II. The solid and dashed curves correspond to P_{CHORUS} and P_{LSND} , respectively. The horizontal solid line denotes the limit of the sensitivity at CHORUS and the horizontal dashed one denotes the lower bound from the recent reported LSND events.

figure 2:

Upper bounds of P_{CHORUS} and P_{LSND} versus Δm_{31}^2 in the case (C) with the hierarchy II. Notations are same as in fig.1.

$\Delta m^2(\text{eV}^2)$	$a_e^{(-)}$	$a_e^{(+)}$	$a_\mu^{(-)}$	$a_\mu^{(+)}$
1	0.011	0.989	0.028	0.972
4	0.042	0.958	0.015	0.985
6	0.036	0.964	0.020	0.980
30	0.039	0.961	0.036	0.964
50	0.039	0.961	0.028	0.972

Table 1: Values of the upper and lower bounds $a_\alpha^{(-)}$ and $a_\alpha^{(+)}$ ($\alpha = e, \mu$) for $U_{\alpha i}^2$

	(A)	(B)	(C)
I ($\Delta m_{31}^2 = 6\text{eV}^2$) P_{CHORUS} P_{LSND} Atmospheric Neutrino Neutrino mass hierarchy	3×10^{-6} 1.9×10^{-3} $\nu_\mu - \nu_\tau$ ordinary	9.8×10^{-4} 5.1×10^{-4} $\nu_\mu - \nu_e$ $\nu_1 - \nu_3$ inverse \times	9.8×10^{-4} 1.9×10^{-3} no $\nu_1 - \nu_2$ inverse
II ($\Delta m_{31}^2 = 6\text{eV}^2$) P_{CHORUS} P_{LSND} Atmospheric Neutrino Neutrino Mass Hierarchy	3×10^{-6} 1.9×10^{-3} $\nu_\mu - \nu_\tau$ $\nu_1 - \nu_3$ inverse \times	9.8×10^{-4} 5.1×10^{-4} $\nu_\mu - \nu_e$ ordinary	9.8×10^{-4} 1.9×10^{-3} no $\nu_2 - \nu_3$ inverse

Table 2: Predicted upper bounds of P_{CHORUS} and P_{LSND} in the case of $\Delta m_{31}^2 = 6\text{eV}^2$, the situation of the atmospheric neutrino anomaly, and the neutrino mass hierarchy for each case. Here, \times denotes the unnatural huge Majorana mass hierarchy.

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